QBSML: Modified Numerals

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Team Semantics and Dependence
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Plan for today

- Questions about Aloni (2021)
- Bivalence and Law of Excluded Middle
- QBSML: Aloni & van Ormondt (2021)
- Modified Numerals
- QBSML and Modified Numerals
- Jialiang on QBSML and monotonic inferences

Bivalence and Law of Excluded Middle

Galliani (2021):

The law of the excluded middle does not hold in dependence logic (just as it does not hold in independence friendly logic): for example, if a team X contains both assignments s with $s(x) = s(y)$ and assignments s' with $s'(x) \neq s'(y)$ then $X \not\models x = y$ and $X \not\models x \neq y$.

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Bivalence: every proposition is true or false
LEM: for every proposition p, p is true or $\neg p$ is true ($p \vee \neg p$)

Bivalence and Law of Excluded Middle (2)

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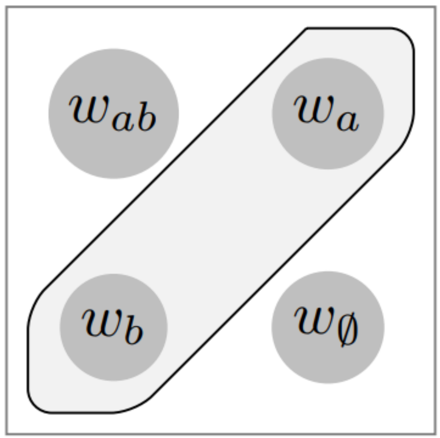
~~$\models NE \vee \neg NE$~~ $(a \vee b)^+ \vee \neg (a \vee b)^+$

In general, LEM is satisfied for classical disjuncts, but BSML clearly does not satisfy bivalence (see example below).



$$M, s \models a \vee \neg a;\ M, s \not\models a;\ M, s \not\models \neg a$$

QBSML

Language:

$t ::= c | v$

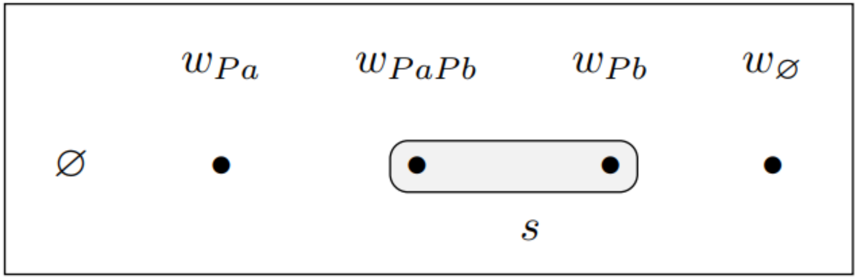
$\phi ::= P^n(\vec{t}) \mid \phi \vee \psi \mid \phi \wedge \psi \mid \exists v \phi \mid \forall v \phi \mid \Box \phi \mid NE$

Model:

$M = \langle W, D, R, I \rangle$

Information State:

A state is set of indices $i = \langle w_i, g_i \rangle$



Empty assignment

Operation on States

What happens when a variable is added to the information state?

Update:

$$g[x/d] := (g \setminus \{\langle x, g(x) \rangle\}) \cup \{\langle x, d \rangle\}$$

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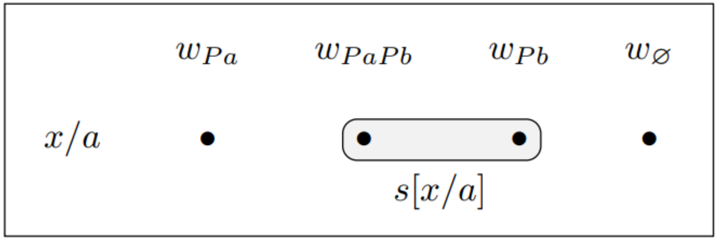
Individual x-extension of a state:

$$s[x/d] := \{i[x/d] | i \in s\}$$

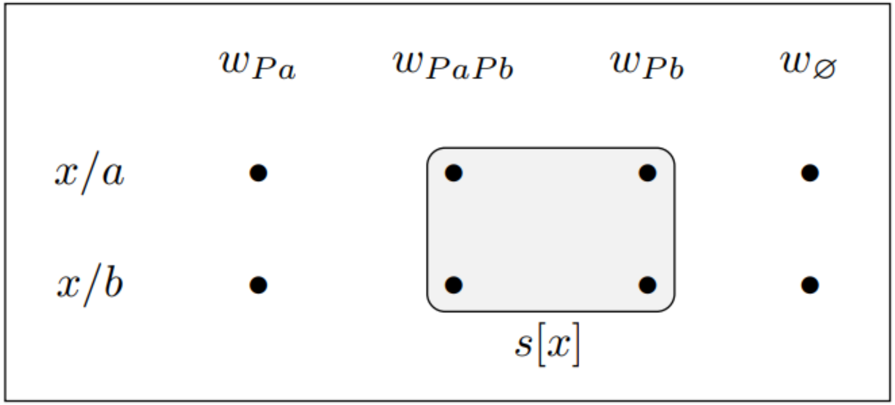


Individual x-extension

Operation on States

Universal x-extension:

$$s[x] := \{i[x/d] | i \in s \;\&\; d \in D\}$$



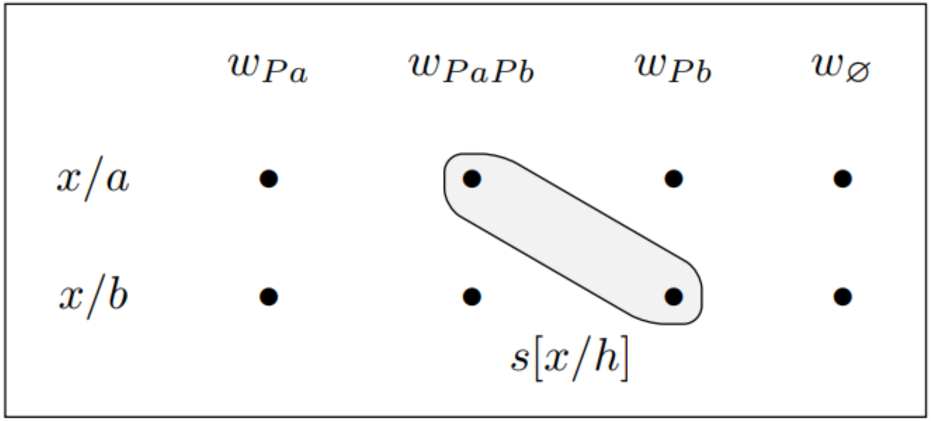
Universal x-extension

Operation on States

Functional *x*-extension:

$$s[x/h] := \{i[x/d] | i \in s \;\&\; d \in h(i)\}$$

$$h : s \mapsto \wp(D) \backslash \varnothing$$



~~Universal~~ *x*-extension
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$$M, s \models \Box\phi \iff \forall i \in s : R(w_i)[g_i] \models \phi$$

$$M, s \models \Diamond\phi \iff \forall i \in s : \exists X \subseteq R(w_i) \text{ and } X \neq \emptyset \text{ and } X[g_i] \models \phi$$

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$$s^{\downarrow} := \{w \in W | \langle w, g \rangle \in s\}$$

R is **state-based** iff $\forall w \in s^{\downarrow} : R(w) = s^{\downarrow}$
R is **indisputable** iff $\forall w, v \in s^{\downarrow} : R(w) = R(v)$

Illustration

$M, s \models \Box\phi \iff \forall i \in s : R(w_i)[g_i] \models \phi$
$M, s \models \Diamond\phi \iff \forall i \in s : \exists X \subseteq R(w_i)$ and $X \neq \varnothing$ and $X[g_i] \models \phi$

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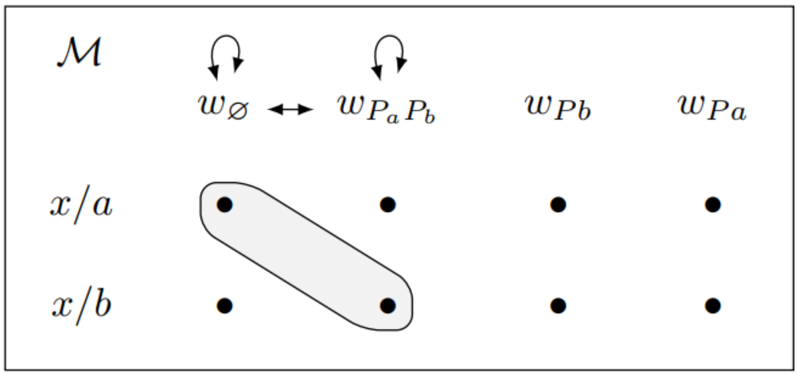

Which statements are correct?

1. R is state-based
2. R is indisputable
3. $M, s \models Px$
4. $M, s \models \Diamond Px$
5. $M, s \models \Box Px$
6. $M, s \dashv \Box Px$

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b. The house has at least three bedrooms.

(2) a. A pentagon has more than 3 sides.

b. ?A pentagon has at least 4 sides.

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Can you think of other examples with the same pattern ?
Nouwen (2010) distinguishes two kinds of numerals, but ignorance effects have probably a different pattern:

Class A: *over n*, *under n*, *between n and m*, . . .

Class B: *minimally*, *up to*, *from n to m*, *or fewer*, . . .

Modified Numerals

Büring (2008) proposes that superlative modified numerals involve **disjunctive meanings**:

at least n $\mapsto \lambda P \lambda Q |P \cap Q| > n \vee |\mathbf{P} \cap \mathbf{Q}| = \mathbf{n}$

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These effects can all be captured in QBSML by assuming the above lexical entry and the results carry over given the operation of pragmatic enrichment on disjunction we saw yesterday.

QBSML and Modified Numerals

Klaus has at least three children. [Ignorance]
$(\texttt{three} \vee \texttt{more})^+ \models \Diamond\texttt{three} \wedge \Diamond\texttt{more}$

Every woman in my family has at least three children. [Obviation]
$(\forall x(\texttt{three}(x) \vee \texttt{more}(x)))^+ \not\models \forall x(\Diamond\texttt{three}(x) \wedge \Diamond\texttt{more}(x))$

Every woman in my family has at least three children. [Distribution$^{\Diamond}$]
$(\forall x(\texttt{three}(x) \vee \texttt{more}(x)))^+ \models \exists x\Diamond\,\texttt{three}(x) \wedge \exists x\Diamond\,\texttt{more}(x)$

You are required to read at least three books. [$\Box$ free choice]
$(\Box(\texttt{three} \vee \texttt{more}))^+ \models \Diamond\,\texttt{three} \wedge \Diamond\,\texttt{more}$

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$(\Diamond(\texttt{three} \vee \texttt{more}))^+ \models \Diamond\,\texttt{three} \wedge \Diamond\,\texttt{more}$

?Klaus does not have at least three children. [Negation]
$(\neg(\texttt{three} \vee \texttt{more}))^+ \models \neg\texttt{three} \wedge \neg\texttt{more}$

Exercise

Aloni (2022) observes that QBSML can model distributive inference under both total and partial information (see examples (44) and (45) in Aloni 2022). Aloni and van Ormondt (2021) prove these results (see proofs in the paper).

Aloni (2022) claims that QSBML can also model *all-others-free-choice* and *all-others-dual-prohibition* readings (see examples (46) and (47) in Aloni 2022). Prove that (46) and (47) are indeed valid in QBSML.

Break

Bisschopsmolen, Maastricht