Team Semantics and Dependence

Marco Degano Maria Aloni

Team Semantics and Dependence
12 January 2022

Plan for today

- Welcome!
- Outline of the project
- Historical overview: Dependence and Quantifiers
- Bilateral State-based Modal Logic (BSML)
- Free Choice
- Free Choice and BSML
- Variants of BSML

Outline of the project

- Week 1:
  - **Tue 11** - Introduction & Free Choice
  - **Wed 12** - QBSML: Modified Numerals & Monotonic Inferences (Jialiang)
  - **Thu 13** - Indefinites: Exceptional Scope & Marked Indefinites
  - **Fri 14** - Overview Presentations & Final Projects
  - **Fri 14, 6 pm** - Deadline exercises
- Week 2:
  - **Mon 17, 11 am** - Deadline Presentation Topic/Reading
  - **Tue 18** - Guest Lecture Aleksi
  - **Wed 19** - Presentations
  - **Thu 20** - Presentations
  - **Fri 21, 6 pm** - Topics/Ideas(/Groups) Final Project
- Week 3:
  - **Mon 24, 13 - 17** - Meetings Final Project
- Week 4:
  - **Fri 4, 6 pm** - Deadline Final Project

Branching Quantifiers

Henkin (1961) introduces **finite partially ordered quantifiers** (aka *branching quantifiers*)

In FOL quantifiers are linearly ordered

$$Qx_1, \ldots, Qx_{n-1}, \mathbf{Qx_n}$$

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Branching quantifiers are quantifiers which can be **partially ordered**:

$$Q_H xyzw\ \phi(x, y, z, w) \equiv \begin{pmatrix} \forall x & \exists y \\ \forall z & \exists w \end{pmatrix} \phi(x, y, z, w)$$

Dependence as (Skolem) function

$$\begin{pmatrix} \forall x & \exists y \\ \forall z & \exists w \end{pmatrix} \phi(x, y, z, w)$$

For every x and z there are y and w, where y *depends only* on x, and w *depends only* on z, such that $\phi(x, y, z, w)$.

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There exist *functions* f and g mapping all of D into D such that for every x and z $\phi(x, \mathbf{f(x)}, z, \mathbf{g(z)})$.

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Enderton, Walkoe, 1970: first-order logic + FPO quantifiers $\equiv$ existential second-order logic (ESO)

Formal Semantics in the 70s

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Hintikka’s *simple* example:

Some relative of each villager and some relative of each townsman hate each other.

Independence Friendly Logic

IF logic is an unfolded version of FPO quantifiers:

$$\begin{pmatrix} \forall x & \exists y \\ \forall z & \exists w \end{pmatrix} \phi(x, y, z, w)$$

$$\forall x \exists y \forall z \exists w / \forall x \, \phi(x, y, z, w)$$

In IF logic (in)dependence relations are expressed by **two factors**: syntactic scope and the independence indicator '/'.

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(Hintikka (1968) founded the framework of *game-theoretical semantics* (quantifiers as strategy *functions*). IF logic allows games of imperfect information.)

Quiz

$$\forall x \exists y \forall z \exists w/\forall x \, \phi(x, y, z, w)$$

Translate using IF quantifiers

Some$_x$ relative of each$_y$ villager and some$_z$ relative of each$_w$ townsman hate each other.

Quiz

Note that other, weaker, readings are possible where branching quantifications is not required. For an overview and references, see Branching Quantification v. Two-way Quantification by Nina Gierasimczuk and Jakub Szymanik (https://www.jakubszymanik.com/papers/HTR.pdf).

Translate using IF quantifiers

Some$_x$ relative of each$_y$ villager and some$_z$ relative of each$_w$ townsman hate each other.

$$\forall y \exists x \forall w \exists z / \forall y \, \text{villager}(y) \wedge \text{townsman}(z) \rightarrow \text{relative}(y)(x) \wedge \text{relative}(w)(z) \wedge \text{hate}(z)(x) \wedge \text{hate}(x)(z)$$

Compositionality

Barwise (1979), Hintikka (1996), and Hintikka and Sandu (1996) claim that IF cannot be compositional:

> *There is no realistic hope of formulating compositional truth-conditions for IF first-order sentences, even though I have not given a strict impossibility proof to that effect. (Hintikka 1996)*

$$\exists u/\forall \mathbf{x}\ \phi(x, y, z, u)$$

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Is Hodges's compositional system for logics of dependence really compositional ? Well, it depends ...

Dependence Logic

In Dependence Logic (Väänänen 2007), dependence is treated **separately** as an atomic statement.

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Let M be a model and T a set of assignments $i : Var \mapsto M$:

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$M, T \models dep(\vec{x}, y)$ iff y **depends** on x in M and T

For all assignments $i, i' \in T$:

$$\bigwedge_{k=1}^{n} i(x_k) = i'(x_k) \text{ then } i(y) = i'(y)$$

T	x	y	z	v
i_1	a	c	a	d
i_2	a	c	a	d
i_3	b	a	b	d
i_4	b	a	d	d

Which statements are wrong?

1. $M, T \models dep(x, y)$
2. $M, T \models dep(x, v)$
3. $M, T \models dep(v, x)$
4. $M, T \models dep(xy, z)$
5. $M, T \models dep(\emptyset, v)$

Let M be a model and T a set of assignments $i : Var \mapsto M$:

$$M, T \models dep(\vec{x}, y) \text{ iff } y \textbf{ depends } \text{on } x \text{ in } M \text{ and } T$$

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3 and 4 are wrong

Semantic Clauses for Propositional Team Logic

- $M, T \models p \Leftrightarrow \forall i \in T : i(p) = 1$
- $M, T \models \phi \wedge \psi \Leftrightarrow M, T \models \phi$ and $M, T \models \psi$
- $M, T \models \phi \vee \psi \Leftrightarrow T = T_1 \cup T_2$ s.t. $M, T_1 \models \phi$ and $M, T_2 \models \psi$.
- $M, T \models NE \Leftrightarrow T \neq \varnothing$

(Other definitions of disjunction can be formalized here, Yang & Väänänen 2016; Aloni 2018)

Quiz

Who wrote the following passage?

> *Sets of assignments S encode several kinds of 'dependence' between variables. There may not be one single intuition. 'Dependence' may mean functional dependence (if two assignments agree in S on x, they also agree on y), but also other kinds of 'correlation' among value ranges.*

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Many frameworks in the formal semantics/philosophical tradition rely on **sets of assignments**: Hans Kamp's DRT, Dynamic Semantics, Alternative Semantics, Inquisitive Semantics, . . .

Bilateral State-based Modal Logic (BSML)

Maria Aloni develops a *Bilateral State-based Modal Logic* (BSML) combining insights from both traditions.

Bilateral systems model **assertion** and **rejection**, rather than truth and falsity.

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In team-based modal logic, a team is a set of possible worlds. In state-based modal logic, teams are interpreted as **information states** capturing the assertability and rejectability of a sentence.

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BSML integrates neglect-zero **cognitive tendencies** via the *NE* atom.

Break

Break

Muiden Castle, Muiden

BSML (Semantic Clauses)

Formulas are interpreted in a model $M = \langle W, R, V \rangle$ with respect to a state $s \subseteq W$ with both support and anti-support conditions:

$$
\begin{array}{rcl}
M, s \models p & \textit{iff} & \forall w \in s : V(w, p) = 1 \\
M, s \dashv p & \textit{iff} & \forall w \in s : V(w, p) = 0 \\
M, s \models \neg\phi & \textit{iff} & M, s \dashv \phi \\
M, s \dashv \neg\phi & \textit{iff} & M, s \models \phi \\
M, s \models \phi \vee \psi & \textit{iff} & \exists t, t' : t \cup t' = s \ \& \ M, t \models \phi \ \& \ M, t' \models \psi \\
M, s \dashv \phi \vee \psi & \textit{iff} & M, s \dashv \phi \ \& \ M, s \dashv \psi \\
M, s \models \phi \wedge \psi & \textit{iff} & M, s \models \phi \ \& \ M, s \models \psi \\
M, s \dashv \phi \wedge \psi & \textit{iff} & \exists t, t' : t \cup t' = s \ \& \ M, t \dashv \phi \ \& \ M, t' \dashv \psi \\
M, s \models \Diamond\phi & \textit{iff} & \forall w \in s : \exists t \subseteq R[w] : t \neq \emptyset \ \& \ M, t \models \phi \\
M, s \dashv \Diamond\phi & \textit{iff} & \forall w \in s : M, R[w] \dashv \phi \\
M, s \models \mathrm{NE} & \textit{iff} & s \neq \emptyset \\
M, s \dashv \mathrm{NE} & \textit{iff} & s = \emptyset
\end{array}
$$

BSML (Accessibility Relation)

Let $M = (W, R, V)$ and $s \subseteq W$.

- R is indisputable in (M, s) iff for all $w, v \in s : R[w] = R[v]$.
- R is state-based in (M, s) iff for all $w \in s : R[w] = s$

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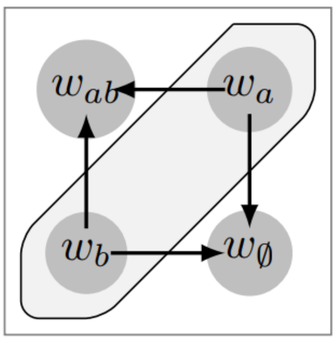
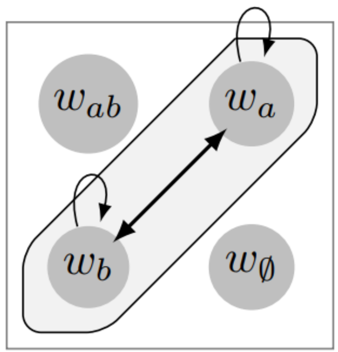
- R is indisputable in (M, s) iff for all $w, v \in s : R[w] = R[v]$.
- R is state-based in (M, s) iff for all $w \in s : R[w] = s$

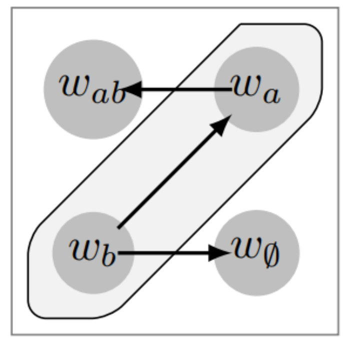
Which one is state-based? Which one indisputable?



INDISPUTABLE, NOT STATE-BASED



STATE-BASED



NEITHER

BSML (Pragmatic Enrichment)

Pragmatic enrichment recursively defined for all NE-free formulas of the language:

$$[\alpha]^+ \equiv \alpha \wedge NE$$

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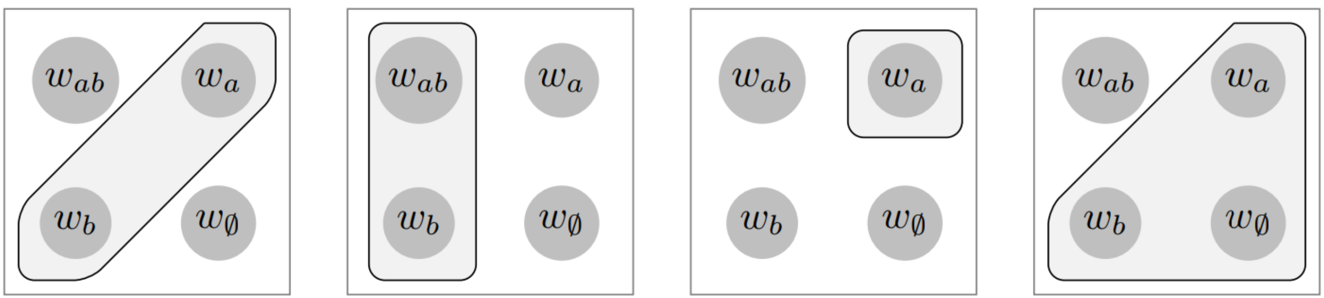
Which states validate $(a \vee b)$? Which ones $(a \vee b)^{+}$?



BOTH

BOTH

ONLY (A v B)

NEITHER

Free Choice

Disjunctions under modals give rise to **free choice inferences**:

(1) a. You may take an apple or a pear. $\Diamond(A \vee B)$

b. You may take an apple. $\Diamond A$

c. You may take a pear. $\Diamond B$

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Different solutions to the problem with both semantic and pragmatic approaches.

Data (1)

Free Choice Disjunctions are compatible with **exclusivity inferences**:

(2) Exclusivity Inference

a. You may take an apple or a pear.

b. $\rightsquigarrow$ You may not take both fruit together.

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a. You may take an apple or a pear.

b. $\rightsquigarrow$ You may not take both fruit together.

The latter are taken to be scalar phenomena, which can be **cancelled**, unlike free choice:

(3) You may take an apple or a pear.

a.# In fact, you may not take an apple.

b. In fact, you may take both.

Data (2)

(4) Deontic vs Epistemic

a. Deontic: John may take an apple or a pear.

b. Epistemic: John might be in the office or in the gym.

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(5) Wide Scope FC

a. John may take an apple or he may take a pear.
$\rightsquigarrow$ John may take an apple and may take a pear.

b. John might be in the office or he might be in the gym.
$\rightsquigarrow$ John might be in the office and might be in the gym.

Data (3)

In downward entailing contexts, different effects arise (e.g. **dual prohibition** under negation):

(6) You may not take an apple or a pear. $\neg\Diamond(A \vee B)$

a. $\approx$ You may not take an apple and you may not take a pear. $\neg\Diamond A \wedge \neg\Diamond B$

b. $\not\approx$ You are not free to choose. $\neg(\Diamond A \wedge \Diamond B)$

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b. $\not\approx$ You are not free to choose. $\neg(\Diamond A \wedge \Diamond B)$

Overt cancellations of FC are possible with deontic modals, but not with epistemic ones:

(7) **Slucing**

a. You are allowed to take an apple or a pear. I don't know which.

b. John might be in Paris or in London. I don't know where.

from (7a) you cannot get FC, while with (7b) the inference is still there

Data (4)

Interaction with **quantifiers** (Chemla 2009):

(8) Universal FC

a. Every student is allowed to have an apple or a pear.
$\forall x \Diamond (Px \vee Qx)$

b. $\rightsquigarrow$ Every student is allowed to have an apple. $\forall x \Diamond Px$

c. $\rightsquigarrow$ Every student is allowed to have a pear. $\forall x \Diamond Qx$

(9) Negative universal FC

a. No student is required to solve both problem A and problem B. $\neg \exists x \Box (Px \wedge Qx)$

b. $\rightsquigarrow$ No student is required to solve A. $\neg \exists x \Box Px$

c. $\rightsquigarrow$ No student is required to solve B. $\neg \exists x \Box Qx$

Data (5)

FC inferences become presuppositions when FC disjunction is embedded in the scope of focus sensitive operators (e.g. *only*, Alxatib 2014):

(10) Are we only allowed to eat [ice cream or cake]$_F$?

a. $\rightsquigarrow$ We are allowed to eat ice cream.

b. $\rightsquigarrow$ We are allowed to eat cake.

(11) Are we allowed to eat ice cream or cake?

a. $\not\rightsquigarrow$ We are allowed to eat ice cream.

b. $\not\rightsquigarrow$ We are allowed to eat cake.

BSML and Free Choice

Wide vs Narrow Free Choice:

- $\Diamond(\alpha \vee \beta)^+ \models \Diamond\alpha \wedge \Diamond\beta$
- $(\Diamond\alpha \vee \Diamond\beta)^+ \models \Diamond\alpha \wedge \Diamond\beta$ [If R is indisputable]

Epistemic vs Deontic Modals

- Epistemic modals: R is state-based
  $\Rightarrow$ **both** narrow and wide scope FC predicted
- Deontic modals: R may be indisputable if speaker is **knowledgable**
  $\Rightarrow$ Narrow FC always predicted.
  $\Rightarrow$ Wide FC only if speaker knows what is permitted.

$\Rightarrow$ FC cancellations involve **always** a wide scope configuration.

Variants of BSML

BSML$^+$: **global pragmatic enrichment** function $[\,]^+$

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Neglect-zero effects can be modelled in different ways:

$\textbf{BSML}^{*}$: BSML without the empty state
$\textbf{BSML}^{\emptyset}$: BSML without the NE atom
$\textbf{BSML}^{lex}$: BSML with local pragmatic enrichment

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Neglect-zero effects can be modelled in different ways:

BSML*: BSML without the empty state
BSML$^\emptyset$: BSML without the NE atom
BSML^{lex}: BSML with local pragmatic enrichment

These may reflect different reasoning styles or lexicalizations of neglect-zero effects:

BSML*: global neglect-zero effects
BSML$^\emptyset$: mathematical reasoning
BSML^{lex}: lexicalization of pragmatic enrichment for e.g. modals.

Variants of BSML (2)

We can study different hypotheses about the availability and robustness of these systems:

			$\mathrm{BSML}^{\emptyset}$	BSML^{lex}	BSML^{*}
NS FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	s	-	+	+
Dual Prohibition	$\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$	s	+	+	+
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$	w	-	-	+
Modal disjunction	$\alpha \vee \beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	w	-	-	+
WS FC	$\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	?	-	-	+

Exercise

At the end of each lecture, you will be asked to do an exercise to test your understanding of the frameworks presented in the readings. You are welcome to do these exercises in groups if you prefer. You should submit your solutions (as a group) in pdf format via email by Friday 14, 6 pm.

Prove that Negative Free Choice is invalid in $BSML^+$ while it is valid in $BSML^*$:

$$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$$